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CALCULUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

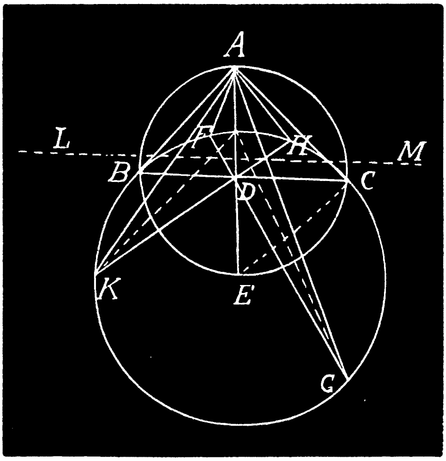
SOLUTIONS TO PROBLEMS.

3. Proposed by H. C. WHITAKER, B. S., M. E., Professor of Mathematics, Manual Training School, Philadelphia, Pennsylvania..

Given: the product of two sides of a triangle is 6000 the length of the bisector of the included angle is 60. Required: What is the maximum area, and what the greatest length of the third side.

II A Geometrical construction by W. B. RIEGNER Philadelphia, Pennsylvania.

Make $AD=60$, $AE=\frac{6000}{60}=1000$, $EC=\surd[DE^2+(AD\times DE)]$. Any triangle with apex A and the third side a chord through D fulfills the conditions. No other triangle can. The centre of the circumscribing circle of each of the triangles is on line LM. The bisectors of each triangle rest on arc BFC. The area swept over by the triangle, the bisector remaining fixed is 13921. Area$=1200\surd6$, 3rd side$=40\surd10$.



6. Proposed by O. S. KIBLER, Superintendent of Schools, West Middleburg, Ohio.

A string is wound spirally twenty times around a cylinder 20 feet high and 2 feet in diameter. Through what distance will a dove fly in unwinding the string keeping it tense at all times (1) flying in the same plane and (2) not flying in the same plane?

II. Solution by G. B. M. ZERR, Principal of Schools, Staunton, Virginia.

Let $ABCD$....be a prism of n sides inscribed in the cylinder $ABCDE$ of radius $OA=r$.

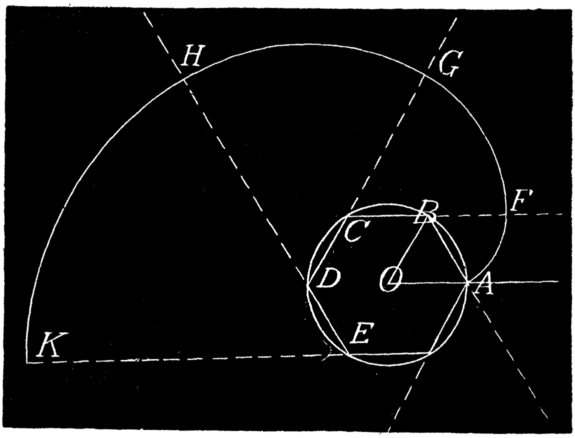
Then $\angle AOB=\angle ABF=\angle BCG=\&c=\frac{2\pi}{n}$ side $AB=2r\sin\frac{\pi}{n}$.

(1) Arc $AFGH$....is a portion of the path of the dove, arc AF is described with radius $AB=2r\sin\frac{\pi}{n}$ and central angle $ABF=\frac{2\pi}{n}$. $\therefore$ length arc $AF=2r\times\frac{2\pi}{n}\sin\frac{\pi}{n}$. Similarly length of arc $FG=4r\times\frac{2\pi}{n}\sin\frac{\pi}{n}$, length of arc $GH=6r\times\frac{2\pi}{n}\sin\frac{\pi}{n}$.

The radius for nth sides of first revolution $=2nr\sin\frac{\pi}{n}$

The radius for nth side of second revolution $=4nr\sin\frac{\pi}{n}$

The radius for nth side of mth revolution $=2mnr\sin\frac{\pi}{n}$



$$\therefore \text{ length of path} = Sec = 2r\times\frac{2\pi}{n}\sin\frac{\pi}{n}\left\{1+2+3 \ldots\ldots +(mn-1)\right\}$$

$$=2r\times\frac{2\pi}{n}\sin\frac{\pi}{n}\times\frac{mn(mn-1)}{2}=2\pi rm(mn-1)\sin\frac{\pi}{n}.$$

Now when n becomes infinite the prism becomes a cylinder and $n\sin\frac{\pi}{n}=\pi r$ also $\sin\frac{\pi}{n}=0$.

$\therefore$ $S=2\pi^2r^2m^2$. In this case $r=1$, $m=20$.

$\therefore$ $S=2(20)^2\pi^2=800\pi^2=7895.72$ feet approximately.

(2) In this case let the string cross the edge of the prism at a distance d apart. Then since there are n sides, one side falls a distance $\frac{d}{n}$ and the length of the string on one face $=\sqrt{4r^2\sin^2\frac{\pi}{n}+\frac{d^2}{n^2}}$ the rest of the notation is the same as in case one since the string is kept horizontal.

$$\therefore \text{ length} = S = \frac{2\pi}{n}\sqrt{4r^2\sin^2\frac{\pi}{n}+\frac{d^2}{n^2}}\left\{1+1+3\ldots\ldots+(mn-1)\right\}$$

$$=\frac{2\pi}{n}\sqrt{4r^2\sin^2\frac{\pi}{n}+\frac{d^2}{n^2}}\,\frac{mn(mn-1)}{2}=\pi m(mn-1)\sqrt{4r^2\sin^2\frac{\pi}{n}+\frac{d^2}{n^2}}$$

$=\pi m^2\sqrt{4\pi^2r^4+d^2}$,when n is infinite but $r=1$ and $m=20$, $d=1$.

$\therefore$ $S=400\pi\sqrt{(4\pi^2+1)}=7995.12$ feet approximately.

[The above revised solution of Professor Zerr's now agrees in result with that of Professor Hume previously published.—Editor.]

17. Proposed by H. W. DRAUGHON, Clinton, Louisiana

Find the volume generated by revolving a circular segment whose base is a given chord ($2a$), about any diameter as an axis.

II. First Solution by F. P MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy, New Windsor College, New Windsor, Maryland.

Let $\angle ECH=A$, $CA=r$, $AE=a$, $CE=x$, and $\angle ACD=\theta$; then $x'=r\cos\theta=\sqrt{(r^2-a^2)}$, and the required volume generated becomes

$$V=2(2\pi x\sin A)\int_{x'}^{r}\sqrt{(r^2-x^2)}dx=4\pi\sin A\left[-\tfrac{1}{3}\left(r^2-x^2\right)^{\frac{3}{2}}\right]_{x'}^{r}=\tfrac{4}{3}\pi a^3\sin A.$$

SECOND SOLUTION.

Put $\angle CAE=\phi$; then $x=r\sin\phi$, $dx=r\cos\phi d\phi$, and $\sqrt{(r^2-x^2)}=r\cos\phi$. The limits of integration are $\frac{1}{2}\pi$ and $\cos^{-1}(a/r)=\phi'$; and after obvious transformations, we have

$$V=4\pi\sin A\int_{\phi'}^{\frac{1}{2}\pi}r^3\cos^2\phi\sin\phi d\phi=4\pi\sin A\left[-\tfrac{1}{3}r^3\cos^3\phi\right]_{\phi'}^{\frac{1}{2}\pi}=\tfrac{4}{3}\pi a^3\sin A.$$

THIRD SOLUTION.

Represent the center of gravity of the segment $ADBE$ by G; then from Mechanics, we have $CG=\dfrac{\frac{1}{12}(2a)^3}{r^2\sin^{-1}(a/r)-a\sqrt{(r^2--a^2)}}$, in which the denominator is the area of the segment $ADBE,=\mathbf{A}$. Since $FG=CG\sin A=(2a^3/3\mathbf{A})\sin A$, the *Centrobaryc Method* gives for the volume generated $V=2\pi(2a^3/3\mathbf{A})\sin A\times\mathbf{A}=\frac{4}{3}\pi a^3\sin A$.

[In the solution by the Proposer, previously published the lower integral should have been $\sqrt{(R^2-a^2)}$. There was a misprint.—Editor.]

Note on Prob. No. 13 by JOHN DOLMAN, Jr., Philadelphia, Pennsylvania.

"It may be worth noting that the solution of No. 13 is incomplete in failing to state that the value of x found makes y a minimum only when angle EAD equals or is less than $\cos^{-1}\frac{4}{5}$ [considering the angle negative when D is west of E]. In all other cases the value, $x=\sin^{-1}\frac{4}{5}$, makes y a maximum and the steamers could *meet* in two possible points. All this, together with the critical value found, may be seen from inspection without any analytical work."

PROBLEMS.

27. Proposed by G. B. M ZERR. A. M. Principal of Schools, Staunton, Virginia.

A runs around the circumference of a circular field with velocity m feet; B starts from the centre with velocity $n>m$ feet to catch A. The straight line joining their positions always passes through the centre. Find the equation to the curve described by B, the distance he runs and the time occupied.